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Abstract: Davig and Leeper (2007) have proposed a condition they call the generalized Taylor principle to rule out indeterminate equilibria in a version of the New Keynesian model, where the parameters of the policy rule follow a Markov-switching process. We show that although their condition rules out a subset of indeterminate equilibria, it does not establish uniqueness of the fundamental equilibrium. We discuss the differences between indeterminate fundamental equilibria included by Davig and Leeper's condition and fundamental equilibria that their condition misses.

JEL classification: bounded solutions, multiple fundamental equilibria, historical dependence

Key words: E52

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Please address questions regarding content to Roger E.A. Farmer, Department of Economics, University of California–Los Angeles, Box 951477, Los Angeles, CA 90095-1477, 310-825-6547, rfarmer@econ.ucla.edu; Daniel F. Waggoner, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, GA 30309-4470, 404-498-8278, daniel.f.waggoner@atl.frb.org; or Tao Zha, Research Department, Federal Reserve Bank of Atlanta, 1000 Peachtree Street, N.E., Atlanta, Georgia 30309-4470, 404-498-8353, tzha@earthlink.net.

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I. Introduction

In a recent paper in this journal, Troy Davig and Eric Leeper (2007) provide a generalization of the Taylor principle, the proposition that central banks can stabilize the economy by raising their interest-rate instrument more than one-for-one in response to higher inflation. The Taylor principle was orginally discussed in the context of a constant parameter new-Keynesian model;[1] Davig and Leeper generalize this principle to a class of forward-looking Markov-switching rational expectations models. Reduced form Markov-switching models have been popular tools for studying a wide range of economic issues (Sims and Zha, 2006), including changes in the monetary transmission mechanism, and the Davig-Leeper extension of the Taylor principle to forward looking environments is important since it permits the analysis of alternative policies within a rational expectations framework.

II. The Regime Switching Model

The New-Keynesian model analyzed by Davig and Leeper has two private sector equations,

$$x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1}) + u_t^D, \tag{1}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t^S, \tag{2}$$

where x_t is output, π_t is inflation, i_t is the nominal interest rate, u_t^D is an aggregate demand shock, and u_t^S is an aggregate supply shock. The variables π_t and i_t are percentage deviations from their steady state values and x_t is the deviation of output from its trend path. Davig and Leeper assume that u_t^D and u_t^S are bounded first order autoregressive processes. To derive closed-form solutions, we simplify their example by setting the autoregressive coefficients to zero which implies that u_t^D and u_t^S are bounded, mean zero, and i.i.d. random variables. Nothing of substance hinges on this assumption and it has the advantage of simplifying our notation.

The policy rule is given by

$$i_t = \alpha_{s_t} \pi_t + \gamma_{s_t} x_t, \tag{3}$$

where s_t is a 2-regime Markov process assuming values in $\{1, 2\}$ with transition matrix $P = (p_{ij})$ for $i, j = 1, 2$, with p_{ij} being the probability that $s_t = j$ given that $s_{t-1} = i$.

[1] See, for example, Woodford (2003) and the references therein.

As in Davig and Leeper, we assume that the fundamental shocks u_t^D and u_t^S are independent of the Markov process s_t.

The private sector block, consisting of Equations (1) and (2), has three regime-independent parameters, σ, β and κ. Uncertain monetary policy is represented by Eq (3), the policy rule. This equation has two regime-dependent parameters, α_{s_t} and γ_{s_t}, that capture the degree to which monetary policy responds to inflation and output.

By substituting the policy rule (3) into Eq (1) and rearranging the terms, the regime-switching new-Keynesian model can be written in matrix form as

$$\Gamma_{s_t} y_t = E_t y_{t+1} + \Psi u_t, \tag{4}$$

where

$$y_t = \begin{bmatrix} \pi_t \\ x_t \end{bmatrix}, \; u_t = \begin{bmatrix} u_t^S \\ u_t^D \end{bmatrix}, \tag{5}$$

$$\Gamma_{s_t} = \begin{bmatrix} \beta^{-1} & -\kappa\beta^{-1} \\ \sigma^{-1}(\alpha_{s_t} - \beta^{-1}) & 1 + \sigma^{-1}(\gamma_{s_t} + \kappa\beta^{-1}) \end{bmatrix}, \text{ and } \Psi = \begin{bmatrix} \beta^{-1} & 0 \\ -\sigma^{-1}\beta^{-1} & 1 \end{bmatrix}. \tag{6}$$

III. The Taylor Principle in a Constant Parameter Model

Before explaining the regime switching model, we present a brief analysis of the role of the Taylor principle in the context of the constant parameter model. Although this model is well known, we review it because the properties of indeterminate equilibria in this familiar context are essential to clarifying the results we will present for the regime switching case.

The constant parameter new-Keynesian model is given by the equation,

$$\Gamma y_t = E_t y_{t+1} + \Psi u_t, \tag{7}$$

where

$$\Gamma = \begin{bmatrix} \beta^{-1} & -\kappa\beta^{-1} \\ \sigma^{-1}(\alpha - \beta^{-1}) & 1 + \sigma^{-1}(\gamma + \kappa\beta^{-1}) \end{bmatrix}.$$

A solution to Eq (7) is a stochastic process, satisfying this equation, that describes how the vector of variables y_t evolves through time. Depending on the values of the parameters there may be one or more solutions. One solution describes y_t as a linear function of the shocks. It is given by the expression,

$$y_t = G u_t \tag{8}$$

where

$$G = \Gamma^{-1}\Psi = \frac{1}{\sigma + \gamma + \kappa\alpha} \begin{bmatrix} \sigma + \gamma & \kappa\sigma \\ -\alpha & \sigma \end{bmatrix}. \tag{9}$$

Following McCallum (1983), we refer to Eq (9) as a minimal state variable solution. Since the fundamental shock u_t is bounded, the minimal state variable solution is also bounded in the sense that there exists a real number N such that

$$||y_t|| < N, \text{ for all } t,$$

where $|| \; ||$ is any well-defined norm. This solution is unique in the class of bounded solutions if and only if all the roots of Γ are outside the unit circle.

Davig and Leeper focus on bounded solutions because local approximations to the underlying dynamic stochastic general equilibrium model remain valid in the neighborhood of a steady state and the boundedness assumption allows one to appeal to approximation theorems which assert that the linear model is approximately valid for small noise. Throughout this paper, we follow Davig and Leeper and consider only solutions that remain bounded in this sense.

For some parameter configurations there may be an infinite set of solutions to Eq (7). When this occurs, each member of the set is said to be an indeterminate equilibrium. The minimal state variable solution is a member of this set but there are other solutions that are serially correlated and add additional volatility to the time paths of the interest rate, output and inflation. Indeterminate solutions have attracted considerable recent attention as a possible explanation for U.S. time series data in the period before the Volcker disinflation of 1979-82.

In recent papers on the empirical importance of indeterminate equilibria, Lubik and Schorfheide (2003, 2004) show how to write an indeterminate solution to the constant parameter new-Keynesian model as a linear combination of the minimal state variable solution and a first order moving average component. When there are eigenvalues of Γ that are less than one in absolute value, the Lubik-Schorfheide indeterminate *fundamental* solutions to Eq (7) have the form[2]

$$y_t = Gu_t + Vw_t, \tag{10}$$

[2]Although our analysis applies to equilibria with both fundamental and non-fundamental components, we focus throughout this paper on indeterminate *fundamental* equilibria by setting the sunspot shock to zero. These bubble-free solutions represent equilibrium responses of agents to economically relevant fundamental shocks.

$$w_t = \Lambda w_{t-1} + M u_t, \tag{11}$$

where the random variable w_t is k-dimensional, M is any $k \times 2$ real matrix, V is $2 \times k$, Λ is $k \times k$, and k is the number of eigenvalues of Γ that are less than one in absolute value. The matrices V and Λ (obtained from the Schur decomposition of Γ) satisfy the condition

$$V\Lambda = \Gamma V. \tag{12}$$

By direct substitution one can verify that our proposed solution, represented by Eqs (10) and (11), does indeed satisfy Eq (7). The following steps establish this result,

$$\begin{aligned} E_t y_{t+1} + \Psi u_t &= E_t \left[G u_{t+1} + V \left(\Lambda w_t + M u_{t+1} \right) \right)] + \Psi u_t, && (13) \\ &= V \Lambda w_t + \Psi u_t, && (14) \\ &= \Gamma V w_t + \Gamma G u_t, && (15) \\ &= \Gamma y_t. && (16) \end{aligned}$$

Eq (13) substitutes Eqs (10) and (11), led by one period, into the right hand side of Eq (7). Taking expectations and using the property that u_{t+1} is a mean zero random variable leads to Eq (14). Eq (15) follows from Eqs (12) and (9) and Eq (16) is established by collecting terms in Γ and applying the definition of a solution, Eq (10).

It is worth drawing attention to two special cases of the Lubik-Schorfheide solutions. First, if all the eigenvalues of Γ are greater than one in absolute value, the minimal state variable solution, Eq (8), is unique. A special case of the new-Keynesian model occurs if $\gamma = 0$. This case has been important in the literature since it leads to a particularly simple statement of the uniqueness condition which is satisfied if $|\alpha| > 1$. When the central bank follows a policy of this kind in which it changes the interest rate by more than one-for-one in response to a change in inflation, the central bank is said to follow the Taylor principle.

A second important special case occurs when Γ has only one eigenvalue less than one in absolute value. In this case Λ is equal to this eigenvalue, and V is its associated eigenvector. An example that has this property is provided by the parameterization $\beta = 0.99$, $\sigma = 1.0$, $\kappa = 0.17$, $\gamma = 0.0$, and $\alpha = 0.92$, For these parameters the matrix

Γ is given by

$$\Gamma = \begin{bmatrix} 1.0101 & -0.1717 \\ -0.0901 & 1.1717 \end{bmatrix} \tag{17}$$

which has real roots of 1.2392 and 0.9426. The eigenvector associated with the eigenvalue of 0.9426 is

$$\begin{bmatrix} -0.9306 \\ -0.3659 \end{bmatrix} \tag{18}$$

and the indeterminate solution is

$$y_t = \begin{bmatrix} 0.8648 & 0.1470 \\ -0.7956 & 0.8648 \end{bmatrix} u_t + \begin{bmatrix} -0.9306 \\ -0.3659 \end{bmatrix} w_t \tag{19}$$

where

$$w_t = 0.9426 w_{t-1} + M u_t. \tag{20}$$

This example is important because Lubik and Schorfheide (2004) have documented that U.S. monetary policy in the period from 1960 to 1979 did not satisfy the Taylor principle and that, as a consequence, the data were generated by an indeterminate equilibrium of this form.

IV. The Generalized Taylor Principle in a Regime-Switching Model

In this section we briefly describe Davig and Leeper's generalized Taylor principle. Analogous to the constant parameter model, the regime-switching model also has a minimal state variable solution,

$$y_t = G_{s_t} u_t \tag{21}$$

where

$$G_{s_t} = \Gamma_{s_t}^{-1} \Psi = \frac{1}{\sigma + \gamma_{s_t} + \kappa \alpha_{s_t}} \begin{bmatrix} \sigma + \gamma_{s_t} & \kappa \sigma \\ -\alpha_{s_t} & \sigma \end{bmatrix}. \tag{22}$$

Since the fundamental shocks are bounded, so is the minimal state variable solution.

The Taylor principle provides a simple rule to ensure uniqueness of equilibrium: it works by guaranteeing that all the roots of a given matrix lie inside the unit circle. One would like to find a similar condition to establish a region of the parameter space for which the minimal state variable solution to Eq (4) is the unique bounded equilibrium. This is a challenging problem since the parameters of the model are functions of the switching variable s_t and these parameters enter multiplicatively with y_t. As a consequence, the regime-switching model is inherently nonlinear.

To address the nonlinearity of the model, Davig and Leeper introduce additional variables that coincide with the original variables in some regimes and they study an expanded model that is linear in these newly defined objects. The new variables are $\pi_{1,t}$, $\pi_{2,t}$, $x_{1,t}$, and $x_{1,t}$, which are random variables with the property

$$\pi_t = \begin{cases} \pi_{1,t} & \text{if } s_t = 1 \\ \pi_{2,t} & \text{if } s_t = 2 \end{cases} \text{ and } x_t = \begin{cases} x_{1,t} & \text{if } s_t = 1 \\ x_{2,t} & \text{if } s_t = 2 \end{cases}. \tag{23}$$

With these new random variables, Davig and Leeper derive the following linear system from the original Markov-switching model Eq (4).[3]

$$BY_t = AE_t Y_{t+1} + Cu_t, \tag{24}$$

where

$$Y_t = \begin{bmatrix} \pi_{1,t} \\ \pi_{2,t} \\ x_{1,t} \\ x_{2,t} \end{bmatrix}, A = \begin{bmatrix} \beta p_{11} & \beta p_{12} & 0 & 0 \\ \beta p_{21} & \beta p_{22} & 0 & 0 \\ \sigma^{-1} p_{11} & \sigma^{-1} p_{12} & p_{11} & p_{12} \\ \sigma^{-1} p_{21} & \sigma^{-1} p_{22} & p_{21} & p_{22} \end{bmatrix}, \tag{25}$$

$$B = \begin{bmatrix} 1 & 0 & -\kappa & 0 \\ 0 & 1 & 0 & -\kappa \\ \sigma^{-1}\alpha_1 & 0 & 1+\sigma^{-1}\gamma_1 & 0 \\ 0 & \sigma^{-1}\alpha_2 & 0 & 1+\sigma^{-1}\gamma_2 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}. \tag{26}$$

Multiplying Eq (24) by A^{-1} transforms this equation into the form of Eq (7) where $\Gamma = A^{-1}B$ and $\Psi = A^{-1}C$. It follows from the results discussed in Section III that Eq (24) has a unique bounded solution if and only if all the eigenvalues of $\Gamma = A^{-1}B$ are greater than one in absolute value. This is an application of Davig and Leeper's generalized Taylor principle to the case when the matrix A is nonsingular. More generally, the principle asserts that all the generalized eigenvalues of (B, A) lie inside the unit circle.

Davig and Leeper show, using Definition (23), that a solution to Eq (24) can be used to construct a solution to the original non-linear system, Eq (4): But this does not establish an equivalence between the two systems. Are there solutions of Eq (4) that cannot be represented as solutions of Eq (24)? This is an important question because the generalized Taylor principle is useful only if uniqueness of a solution to

[3]Note that (78) and (79) in Appendix B of Davig and Leeper (2007, page 631) are the same as Eq (4) in this comment. By substituting (80) through (85) into (78) and (79), they rewrite the system in the expanded linear form given by (86) on page 632.

the expanded linear system implies that equilibrium in the original model is also unique. The following section establishes that is not the case by presenting a counter example.

V. A Counterexample to the Generalized Taylor Principle

The generalized Taylor principle is a statement about the set of solutions to the expanded linear model and, applied to this model, the statement is correct. As discussed in Section IV, the Davig and Leeper paper proceeds however as if the original Markov-switching system and the derived linear system were the same.[4]

The purpose of this section is to establish our claim that these two systems are not the same by constructing an example in which the generalized Taylor principle holds and hence the expanded linear model has a unique solution, but the original Markov-switching system has a continuum of fundamental equilibria. Our example is based on the parameterization $\beta = 0.99$, $\sigma = 1.0$, and $\kappa = 0.17$. These parameters are taken from baseline case in Davig and Leeper (2007, page 616). We choose $\gamma_1 = \gamma_2 = 0$, $p_{11} = 0.8$, and $p_{22} = 0.95$ as in the example used to construct Davig and Leeper's Figure 2 on page 617. We consider the two policy regimes represented by $\alpha_1 = 3.0$ and $\alpha_2 = 0.92$, where the parameter values in the second regime are the same as those used to construct an indeterminate example of the constant parameter model in Section III of this comment. Substituting our chosen values into the expressions for the matrices A and B from Eqs (25) and (26) one obtains the following values for the absolute values of the eigenvalues of $A^{-1}B$

$$\begin{bmatrix} 1.5883 \\ 1.5883 \\ 1.2349 \\ 1.0167 \end{bmatrix}. \tag{27}$$

Since these all are greater than 1, Davig and Leeper's generalized Taylor principle implies a unique bounded equilibrium of the expanded linear system, Eq (24). But

[4]See the text at the bottom of second column on page 616 of Davig and Leeper (2007) which states: "The system to be solved consists of (26) and (32). To specify the system whose eigenvalues determine whether there exists a unique bounded equilibrium, we follow the procedure for the Fisherian model." Note that (26) and (32) represent the original Markov-switching system. The paper proceeds to derive the generalized Taylor principle; that is, a set of necessary and sufficient conditions for existence of a unique bounded equilibrium to the expanded linear system (35) on page 617.

it does not imply that if this principle is satisfied the economic model, Eq (4), has a unique equilibrium, as the following example demonstrates.

Using the parameter values introduced above, the matrix Γ_2 from Eq (4) has an eigenvalue of 0.9426 and an associated eigenvector of

$$\begin{bmatrix} -0.9306 \\ -0.3659 \end{bmatrix}. \tag{28}$$

Notice that this eigenvalue of 0.9426 is less than $p_{22} = 0.95$ and consider the following equations that represent our candidate fundamental equilibrium,

$$y_t = G_{s_t} u_t + V w_t, \tag{29}$$

$$w_t = \Lambda_{s_t} w_{t-1} + M_{s_t, s_{t-1}} u_t, \tag{30}$$

where

$$V = \begin{bmatrix} -0.9306 \\ -0.3659 \end{bmatrix},\ \Lambda_{s_t} = \begin{cases} 0 & \text{if } s_t = 1 \\ \frac{0.9426}{0.95} & \text{if } s_t = 2 \end{cases},\ M_{s_t, s_{t-1}} = \begin{cases} 0 & \text{if } s_t = 1 \\ M_{2, s_{t-1}} u_t & \text{if } s_t = 2 \end{cases},$$

and $M_{2,s_{t-1}}$ is any 1×2 real matrix that may or may not depend on s_{t-1}. Notice that $|\Lambda_i| < 1$ in both regimes and hence y_t is bounded. Importantly, $|\Lambda_2|$ is equal to the smallest root of Γ_2 divided by p_{22} and the fact that it is less than one follows from our parameterization in which the smallest root of Γ_2 equals 0.9426 and p_{22} is equal to 0.95. Notice that the form of our solution is close to those for the constant parameter case with the important difference that the parameter matrices G and Λ are different in different regimes. We chose to present a solution of this form because it is directly comparable to the form of solution we derive for the expanded linear system in Section VI. In that section we discuss further the relationship between the two cases.

The following argument establishes that Eqs (29) and (30) do indeed define a solution to (4).

$$E_t y_{t+1} + \Psi u_t = p_{s_t,1} E_t[y_{t+1} | s_{t+1} = 1] + p_{s_t,2} E_t[y_{t+1} | s_{t+1} = 2] + \Psi u_t \tag{31}$$

$$= p_{s_t,1} V \Lambda_1 w_t + p_{s_t,2} V \Lambda_2 w_t + \Psi u_t \tag{32}$$

$$= p_{s_t,2} V \Lambda_2 w_t + \Psi u_t, \tag{33}$$

$$= \Gamma_{s_t} V w_t + \Gamma_{s_t} G_{s_t} u_t \tag{34}$$

$$= \Gamma_{s_t} y_t. \tag{35}$$

Eq (31) decomposes $E_t y_{t+1}$ using the law of iterated expectations. To obtain Eq (32), substitute Eqs (29) and (30), led one period, into the right hand side of Eq (31) and take expectations, using the fact that u_{t+1} has zero mean and is independent $\{s_{t+1}, s_t, \ldots\}$. Eq (33) follows from the fact that $\Lambda_1 = 0$. To obtain Eq (34) notice that if $s_t = 1$, then $p_{12} V \Lambda_2 w_t = \Gamma_1 V w_t$ because $w_t = 0$ and if $s_t = 2$ then $p_{22} V \Lambda_2 w_t = \Gamma_2 V w_t$ because V is the eigenvector of Γ_2 corresponding to the eigenvalue $p_{22}\Lambda_2$. This establishes that $p_{s_t,2} V \Lambda_2 w_t = \Gamma_{s_t} V w_t$. The second term of Eq (34) follows from the construction of the fundamental solution, Eq (21), $\Psi = \Gamma_{s_t} G_{s_t}$. The final line follows from collecting terms in Γ_{s_t} and substituting for y_t from Eq (29).

This argument establishes that Eqs (29) and (30) define a valid bounded solution to (4). More generally, such a construction can be performed whenever Γ_i has an eigenvalue less than p_{ii} in absolute value. In such cases, therefore, even if a cental banker follows a policy that is consistent with Davig and Leeper's generalized Taylor principle, the economy may still be subject to indeterminacy.

VI. Comparing Different Indeterminate Equilibria

Section V demonstrates that the generalized Taylor principle rules out some, but not all indeterminate fundamental equilibria. How do solutions ruled out by this principle differ from those that are not? In this section we show how to write indeterminate solutions to the regime-switching model included by Davig and Leeper's approach and we compare them to those fundamental equilibria presented in Section V.

If we assume that both A and B, as given by Eqs (25) and (26), are invertible and if there are $k \geq 1$ eigenvalues of $\Gamma = A^{-1}B$ that are less than one in absolute value, we may use the analysis of Section III, to write bounded indeterminate solutions to Eq (24) in the form

$$Y_t = B^{-1} C u_t + V w_t, \tag{36}$$

$$w_t = \Lambda w_{t-1} + M_{s_t, s_{t-1}} u_t, \tag{37}$$

where recall that $Y_t = (\pi_{1,t}, \pi_{2,t}, x_{1,t}, x_{2,t})'$, $u_t = (u_t^S u_t^D)'$, and the term $B^{-1}C$ is given by

$$B^{-1}C = \begin{bmatrix} \frac{\sigma+\gamma_1}{\sigma+\gamma_1+\kappa\alpha_1} & \frac{\kappa\sigma}{\sigma+\gamma_1+\kappa\alpha_1} \\ \frac{\sigma+\gamma_2}{\sigma+\gamma_2+\kappa\alpha_2} & \frac{\kappa\sigma}{\sigma+\gamma_2+\kappa\alpha_2} \\ \frac{-\alpha_1}{\sigma+\gamma_1+\kappa\alpha_1} & \frac{\sigma}{\sigma+\gamma_1+\kappa\alpha_1} \\ \frac{-\alpha_2}{\sigma+\gamma_2+\kappa\alpha_2} & \frac{\sigma}{\sigma+\gamma_2+\kappa\alpha_2} \end{bmatrix}. \tag{38}$$

V is a $4 \times k$ matrix, Λ is a $k \times k$ matrix and all the eigenvalues of Λ are less than one in absolute value. The matrices V and Λ satisfy the restriction $V\Lambda = \Gamma V$ and are obtained from the Schur decomposition of Γ. Note that the fundamental shocks come from $\{s_t, s_{t-1}, \ldots u_t, u_{t-1}, \cdots\}$, and thus the solution defined by Eqs (36) and (37) depends only on fundamentals.

The term $M_{s_t,s_{t-1}}$ represents any $k \times 2$ real matrix that may depend on both the current and past regimes. Because u_t is mean zero and independent of $\{s_t, s_{t-1}, \ldots\}$, $E_{t-1}\left(M_{s_t,s_{t-1}} u_t\right) = 0$ and it is straightforward to verify that Eqs (36) and (37) are bounded solutions to the expanded linear system (24). The proof is the same as that given for the constant parameter case in the paragraph following Eq (12).

We have shown how to construct indeterminate solutions to Davig and Leeper's expanded linear system; but this leaves open the question: Can one use bounded indeterminate solutions to Eq (24) to construct solutions to the original nonlinear system? To answer this question define the matrices Π_1, and Π_2 as follows

$$\Pi_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } \Pi_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{39}$$

and let

$$V_{s_t} = \Pi_{s_t} V. \tag{40}$$

Now premultiply Eq (36) by Π_{s_t} and write it as two separate subsystems for $s_t = \{1, 2\}$,

$$y_t = G_{s_t} u_t + V_{s_t} w_t, \tag{41}$$

$$w_t = \Lambda w_{t-1} + M_{s_t,s_{t-1}} u_t, \tag{42}$$

where it follows from Eqs (6), (22), and (38), that

$$G_{s_t} = \Gamma_{s_t}^{-1} \Psi = \Pi_{s_t} B^{-1} C. \tag{43}$$

Notice that these equations are similar to those that we presented in our counter example, with the exception that the parameter matrix, Λ, is the same in both regimes.

This restriction is key to understanding the difference between Davig and Leeper's indeterminate solutions, that are ruled out by the generalized Taylor principle, and solutions such as those presented in our counter-example, that are not.

A comparison of Eqs (29) and (30) with (41) and (42) establishes that these two kinds of solutions have the first term in common, represented by $G_{s_t} u_t$, but differ in the second term since Eq (29) restricts the matrix V_{s_t} to be the same across regimes while Eq (42) restricts Λ_{s_t} to be the same across regimes. Both solutions allow y_t to depend on the current and past policy regimes as well as on the current and past demand and supply shocks and all of the equilibria that we consider in this comment are driven purely by fundamentals.

The main difference between the solution given by Eqs (29) and (30) and those given by Eqs (41) and (42) is the nature of the persistence of the process w_t. In Eqs (41) and (42) the persistence, which is governed by Λ, is independent of the regime, while in Eqs (29) and (30), it can vary across regimes. Indeed, in a related paper, Farmer, Waggoner, and Zha (2007) show that there exist general forms of indeterminate equilibria that include as special cases both the solution represented by Eqs (29) and (30) and the solution represented by Eqs (41) and (42).

We have demonstrated that the Davig-Leeper generalized Taylor principle does not exclude all bounded fundamental solutions to the original economic model. While the Davig-Leeper solutions are interesting, we see no economic reason to prefer one subset of fundamental equilibria over another. The Davig-Leeper solutions restrict Λ to be independent of regime whereas the complete class allows this matrix to be different in different regimes.

VII. Conclusion

In the conventional linear new-Keynesian model, there may be bounded equilibria in addition to the minimal state variable equilibrium for some parameter configurations. These indeterminate equilibria are serially correlated and add additional volatility to the time paths of the interest rate, output and inflation. Since the influential empirical papers of Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004), such equilibria represent leading candidate explanations for U.S. time series data in the period before the Volcker disinflation of 1979-82.

The advent of Markov-switching models makes it possible to describe the periods before and after 1980 as a single rational expectations model and it leads to the

question: Were indeterminate equilibria responsible for the persistence and volatility observed in the post-war U.S. time series data before 1980 after one accounts for the possibility that agents rationally anticipated the possibility of future regime change?

To answer this question one would need to partition the parameter space into two subsets: one associated with indeterminacy and the other with a unique equilibrium. The parameter space in this more complete model includes not only the private sector and policy parameters in each regime, but also the probabilities of switching. Davig and Leeper (2007)'s generalized Taylor principle takes a step towards answering this question. A complete partition of the parameter space into determinate and indeterminate regions remains an important but challenging open question.

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